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# 倒向随机微分超前方程适应解的稳定性

周会会

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**摘 要**: 本文研究了倒向随机微分超前方程（超前 BSDE）适应解的稳定性问题，从理论上证明了在 Lipschitz 条件下超前 BSDE 的适应解具有稳定性。

**关键词**: 倒向随机微分超前方程; 适应解; 稳定性

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# Stability of Adaptive Solutions for Backward Stochastic Differential Equations

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**Abstract:** This paper focuses on the stability of adaptive solutions for backward stochastic differential anticipated equations (advanced BSDE). It is proved theoretically that the adaptive solutions of anticipated BSDE are stable under Lipschitz conditions.

**Keywords:** Anticipated backward stochastic differential equations; adapted solutions; stability

## 1 引 言

Pardoux 和 Peng[1]引入了非线性倒向随机微分方程（BSDE），并证明了在 Lipschitz 条件下非线性 BSDE 适应解的存在唯一性。BSDE 在随机控制、偏微分方程、数理金融、经济等领域都有着广泛的应用，吸引了众多学者对其研究[2-6]。

Peng 和 Yang[7]引入了一类新的倒向随机微分方程，即倒向随机微分超前方程（超前 BSDE），形式如下：

$$\begin{cases} -dY_t = f\left(t, Y_t, Z_t, Y_{t+\delta(t)}, Y_{t+\zeta(t)}\right)\mathrm{d}t - Z_t \mathrm{d}W_t, \quad t\in[0,T], \\ Y_t=\xi_t, \quad t\in[T,T+K], \\ Z_t=\eta_t, \quad t\in[T,T+K] \end{cases} \tag{1}$$

其中$\delta(\cdot)$和$\xi(\cdot)$是定义于[0，$T$]上取值于$R^+$上的两个连续函数且满足下列条件(1)和(2)，

(1) 存在常数$K\geqq 0$ $K\geq 0$，使得$\forall s\in[0,T]$，有

$$s+\delta(s)\le T+K,\ s+\zeta(s)\le T+K.$$

(2) 存在常数$L\geqq 0$ $L\geq 0$，使得$\forall t\in[0,T]$及非负可积函数$g(\cdot)$，有

$$\int_t^T g(s+\delta(s))\mathrm{d}s \le L\int_t^{T+K} g(s)\mathrm{d}s,\ \int_t^T g(s+\zeta(s))\mathrm{d}s \le L\int_t^{T+K} g(s)\mathrm{d}s.$$

并证明了在 Lipschitz 条件下超前 BSDE 适应解的存在唯一性，参见引理 1。

令$(\Omega,F,P)$表示完备的概率空间，假设$(W(t))_{t\in[0,T]}$是概率空间$(\Omega,F,P)$中的$d$维布朗运动，$\{F_t\}_{t\in[0,T]}$是由$W$产生的自然代数流。文中用到下面的概率空间，$L^2(F_t;R^d)$表示$R^d$值的$F_t$-可测的随机变量且满足$E[|\xi|^2]<\infty$. $L_F^2\left(0,T;R^d\right)$表示$R^d$值的$F_t$-适应的随机过程且满足$E\left[\int_0^T|\varphi_t|^2\,\mathrm{d}t\right]<\infty\cdot S_F^2\left(0,T;R^d\right)$表示$L_F^2\left(0,T;R^d\right)$中的连续过程且满足$E\left[\sup_{t\in[0,T]}|\varphi_t|^2\right]<\infty\cdot$

设$f(s,\omega,y,z,\xi,\eta):\Omega\times R^m\times R^{m\times d}\times L^2(F_r;R^m)\times L^2(F_{r'};R^{m\times d})\rightarrow L^2(F_s;R^m)$，对于任意的$s\in[0,T]$，其中$r,r'\in[s,T+K]$，$f$满足条件(H1)和(H2)。

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(H1) 存在常数 C>0，使得 $\forall s\in[0,T]$， $y,y'\in R^m$， $z,z'\in R^{m\times d}$， $\xi_\cdot,\xi'_\cdot\in L_F^2(s,T+K;R^m)$， $\eta_\cdot,\eta'_\cdot\in L_F^2(s,T+K;R^{m\times d})$， $r,\bar{r}\in[s,T+K]$，有 $\left|f(s,y,z,\xi_r,\eta_{\bar{r}})-f(s,y',z',\xi'_r,\eta'_{\bar{r}})\right|\le C\left[|y-y'|+|z-z'|+E^{F_s}\left(|\xi_r-\xi'_r|+|\eta_{\bar{r}}-\eta'_{\bar{r}}|\right)\right]$.

(H2) $E\left[\int_0^T\left|f(s,0,0,0,0)\right|^2\mathrm{d}s\right]<\infty.$

**引理 1**[7] 设 $f$ 满足(H1)和(H2)，$\delta$、$\zeta$ 满足(1)和(2)，则对任意给定的终端条件 $\xi\in S_F^2(T,T+K;R^m)$，$\eta\in L_F^2(T,T+K;R^{m\times d})$，超前 BSDE(1) 有唯一解，即存在唯一一对 $F_t-$ 适应过程 $(Y_\cdot,Z_\cdot)\in S_F^2(0,T+K;R^m)\times L_F^2(0,T+K;R^{m\times d})$ 满足方程(1)。

## 2 超前 BSDE 适应解的稳定性

Hu 和 Peng[8]中给出了 BSDE 适应解的稳定性定理，考虑到超前 BSDE 存在唯一适应解，下面来讨论超前 BSDE 适应解的稳定性。

考虑下面具有参数 $\varepsilon\ge 0$ 的超前 BSDEs，

$$Y_t^\varepsilon=\xi_T^\varepsilon+\int_t^T f^\varepsilon(s,Y_s^\varepsilon,Y_{s+\delta(s)}^\varepsilon,Z_s^\varepsilon,Z_{s+\zeta(s)}^\varepsilon)ds-\int_t^T Z_s^\varepsilon dW_s,\quad t\in[0,T]\tag{2}$$

当 $\varepsilon\to 0$ 时，我们做如下假设，

(H3) $E\left[\left|\xi_T^\varepsilon-\xi_T^0\right|^2\right]\to 0.$

(H4) $E\left[\int_T^{T+K}\left|\xi_s^\varepsilon-\xi_s^0\right|^2ds\right]\to 0,E\left[\int_T^{T+K}\left|\eta_s^\varepsilon-\eta_s^0\right|^2ds\right]\to 0.$

(H5) $\forall t\in[0,T]$，有

$$E\left\{\left|\int_t^T\left[f^\varepsilon\left(s,Y_s^0,Y_{s+\delta(s)}^0,Z_s^0,Z_{s+\zeta(s)}^0\right)-f^0\left(s,Y_s^0,Y_{s+\delta(s)}^0,Z_s^0,Z_{s+\zeta(s)}^0\right)\right]ds\right|^2\right\}\to 0.$$

**定理 1** 假设超前 BSDE 满足(H3)，(H4)和(H5)，我们有

$$\forall t\in[0,T],E\left[\left|Y_t^\varepsilon-Y_t^0\right|^2\right]+E\left[\int_0^T\left|Z_t^\varepsilon-Z_t^0\right|^2dt\right]\to 0,\varepsilon\to 0.$$

**证明：** 令 $\hat{Y}^\varepsilon=Y^\varepsilon-Y^0,\hat{Z}^\varepsilon=Z^\varepsilon-Z^0,\hat{\xi}^\varepsilon=\xi^\varepsilon-\xi^0$，那么当 $t\in[0,T]$ 时，有

$$\begin{aligned}&\hat{Y}_t^\varepsilon+\int_t^T\hat{Z}_s^\varepsilon dW_s\\&=\hat{\xi}_T^\varepsilon+\int_t^T\left[f^\varepsilon(s,Y_s^\varepsilon,Y_{s+\delta(s)}^\varepsilon,Z_s^\varepsilon,Z_{s+\zeta(s)}^\varepsilon)-f^\varepsilon(s,Y_s^0,Y_{s+\delta(s)}^0,Z_s^0,Z_{s+\zeta(s)}^0)\right]ds\\&\quad+\int_t^T\left[f^\varepsilon(s,Y_s^0,Y_{s+\delta(s)}^0,Z_s^0,Z_{s+\zeta(s)}^0)-f^0(s,Y_s^0,Y_{s+\delta(s)}^0,Z_s^0,Z_{s+\zeta(s)}^0)\right]ds.\end{aligned}$$

两边平方并取期望可得

$$\begin{aligned}&E\left[\left|\hat{Y}_t^\varepsilon\right|^2\right]+E\left[\int_t^T\left|\hat{Z}_s^\varepsilon\right|^2ds\right]\\&\le 2E\left[\left|\hat{\xi}_T^\varepsilon+\int_t^T\left[f^\varepsilon\left(s,Y_s^0,Y_{s+\delta(s)}^0,Z_s^0,Z_{s+\zeta(s)}^0\right)-f^0\left(s,Y_s^0,Y_{s+\delta(s)}^0,Z_s^0,Z_{s+\zeta(s)}^0\right)\right]ds\right|^2\right]\\&\quad+6C^2(T-t)E\left\{\int_t^T\left[\left|\hat{Y}_s^\varepsilon\right|^2+\left|\hat{Z}_s^\varepsilon\right|^2+\left(E^{F_s}|\hat{Y}_{s+\delta(s)}^\varepsilon+\hat{Z}_{s+\zeta(s)}^\varepsilon|\right)^2\right]ds\right\}\\&\le 2E\left[\left|\hat{\xi}_T^\varepsilon+\int_t^T\left[f^\varepsilon\left(s,Y_s^0,Y_{s+\delta(s)}^0,Z_s^0,Z_{s+\zeta(s)}^0\right)-f^0\left(s,Y_s^0,Y_{s+\delta(s)}^0,Z_s^0,Z_{s+\zeta(s)}^0\right)\right]ds\right|^2\right]\\&\quad+6C^2(T-t)E\left\{\int_t^T\left[\left|\hat{Y}_s^\varepsilon\right|^2+\left|\hat{Z}_s^\varepsilon\right|^2+2\left|\hat{Y}_{s+\delta(s)}^\varepsilon\right|^2+2\left|\hat{Z}_{s+\zeta(s)}^\varepsilon\right|^2\right]ds\right\}.\end{aligned}$$

注意到

$$\int_t^T\left|\hat{Y}_{s+\delta(s)}^\varepsilon\right|^2ds\le L\int_t^{T+K}\left|\hat{Y}_s^\varepsilon\right|^2ds=L\int_t^T\left|\hat{Y}_s^\varepsilon\right|^2ds+L\int_T^{T+K}\left|\hat{\xi}_s^\varepsilon\right|^2ds.$$

$$\int_t^T\left|\hat{Z}^{\varepsilon}_{s+\delta(s)}\right|^2ds\le L\int_t^{T+K}\left|\hat{Z}^{\varepsilon}_s\right|^2ds=L\int_t^T\left|\hat{Z}^{\varepsilon}_s\right|^2ds+L\int_T^{T+K}\left|\hat{\eta}^{\varepsilon}_s\right|^2ds.$$

则

$$\begin{aligned}&E\left[\left|\hat{Y}^{\varepsilon}_t\right|^2\right]+E\left[\int_t^T\left|\hat{Z}^{\varepsilon}_s\right|^2ds\right]\\&\le 2E\left\{\left|\hat{\xi}^{\varepsilon}_T+\int_t^T\left[f^{\varepsilon}\left(s,Y^0_s,Y^0_{s+\delta(s)},Z^0_s,Z^0_{s+\zeta(s)}\right)-f^0\left(s,Y^0_s,Y^0_{s+\delta(s)},Z^0_s,Z^0_{s+\zeta(s)}\right)\right]ds\right|^2\right\}\\&\quad+12LC^2(T-t)E\left\{\int_T^{T+K}\left[\left|\hat{\xi}^{\varepsilon}_s\right|^2+\left|\hat{\eta}^{\varepsilon}_s\right|^2\right]ds\right\}+6C^2(T-t)(2L+1)E\left\{\int_t^T\left[\left|\hat{Y}^{\varepsilon}_s\right|^2+\left|\hat{Z}^{\varepsilon}_s\right|^2\right]ds\right\}.\end{aligned}$$

令

$$\begin{aligned}C^{\varepsilon}(t)=2E&\left\{\left|\hat{\xi}^{\varepsilon}_T+\int_t^T\left[f^{\varepsilon}\left(s,Y^0_s,Y^0_{s+\delta(s)},Z^0_s,Z^0_{s+\zeta(s)}\right)-f^0\left(s,Y^0_s,Y^0_{s+\delta(s)},Z^0_s,Z^0_{s+\zeta(s)}\right)\right]ds\right|^2\right\}\\&+12LC^2(T-t)E\left\{\int_T^{T+K}\left[\left|\hat{\xi}^{\varepsilon}_s\right|^2+\left|\hat{\eta}^{\varepsilon}_s\right|^2\right]ds\right\}.\end{aligned}$$

那么

$$E\left[\left|\hat{Y}^{\varepsilon}_t\right|^2\right]+E\left[\int_t^T\left|\hat{Z}^{\varepsilon}_s\right|^2ds\right]\le C^{\varepsilon}(t)+6C^2(T-t)(2L+1)E\left\{\int_t^T\left[\left|\hat{Y}^{\varepsilon}_s\right|^2+\left|\hat{Z}^{\varepsilon}_s\right|^2\right]ds\right\}.$$

对于 $t\in[T-\delta,T]$，令 $\delta=\dfrac{1}{12C^2(2L+1)}$，我们有

$$E\left[\left|\hat{Y}^{\varepsilon}_t\right|^2\right]+\frac{1}{2}E\left[\int_t^T\left|\hat{Z}^{\varepsilon}_s\right|^2ds\right]\le C^{\varepsilon}(t)+\frac{1}{2}\int_t^TE\left[\left|\hat{Y}^{\varepsilon}_s\right|^2\right]ds.$$

从而由 Gronwall 不等式，可得

$$E\left[\left|\hat{Y}^{\varepsilon}_t\right|^2\right]\le C^{\varepsilon}(t)\exp\left[\frac{1}{2}(T-t)\right],t\in[T-\delta,T].$$

又由于方程(2)满足(H3)，(H4)和(H5)，从而 $C^{\varepsilon}(t)$ 在 $0\le t\le T$ 上关于 $\varepsilon$ 是一致有界的，因此 $\forall t\in[T-\delta,T],E\left[\left|\hat{Y}^{\varepsilon}_t\right|^2\right]+E\int_t^T\left[\left|\hat{Z}^{\varepsilon}_s\right|^2\right]ds\to 0,\ \varepsilon\to 0$。特别地，当 $\varepsilon\to 0$ 时，$E\left[\left|\hat{Y}^{\varepsilon}_{T-\delta}\right|^2\right]\to 0$。然后用同样的方法可证明在区间 $[T-2\delta,T-\delta]$，$[T-3\delta,T-2\delta],\cdots$ 上结果亦成立。

## 3 结 论

考虑到具有 Lipschitz 条件的超前 BSDE 存在唯一适应解，在此基础上，进一步研究了超前 BSDE 适应解的稳定性问题，从理论上得到了超前 BSDE 的适应解具有稳定性，对超前 BSDE 有了进一步的认识。

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